International Journal of Engineering Sciences & Research Technology

(A Peer Reviewed Online Journal)
Impact Factor: 5.164

IJESRT

ISSN: 2277-9655
Impact Factor: 5.164
CODEN: IJESS7

IJESRT
INTERNATIONAL JOURNAL OF ENGINEERING SCIENCES & RESEARCH TECHNOLOGY

NON-ABELIAN CALIBRATION FIELD THEORY ON SU(2) LIE GROUP WITH YANG-MILLS APPROXIMATION IN 2+1 DIMENSIONS WITH TOPOLOGICAL MASS GENERATION.

A. Alatorre
Department of physics, University of Guadalajara

DOI: 10.5281/zenodo.1502424

ABSTRACT

Calibration field theory on local non-abelian Lie group SU(2) has been developed. Gauge-Lorenz invariance has been used. It is also shown that field action and its field equations may be built in terms of Yang-Mills approximation to obtain massive field terms. Chern-Simmons formalism aloud induced topology in field theory, obtaining at last topological mass generation.

Keywords- Calibration theory, non-abelian field theory, Yang-Mills theory, topological field theory, SU(2) representation, topological mass, Chern-Simmons application, 2+1 field dimensions, quantum cosmology.

1. INTRODUCTION

In modern research [1] it is shown thatChern-Simmons term describes equally the almost all theories of gravitation of impair dimension where local symmetry group result into a cosmological constant value for which values are not infinity. Non-abelian theories may be constructed whenever its local Lie group allows re-normalization conditions to further advances in theory and field equations solving. In standard relativistic formulations, an abelian 2+1 field theory it's a trivial theory because there are no wave propagations [??], however, when calibration transform comes from non-abelian formulation, Chern-Simmons term will not be trivial, and theory itself may be not trivial. Therefore, it is possible to build a quantum approach to gravity from this stand [1,2].

1.1 Motivation

Yang-Mills and Chern-Simmons field action is an analogy ofMaxwell-Chern-Simmons theory over abelian symmetry local groups. One considered thatChern-Simmons action over non-commutative Lie groups is basically a case of Yang-Mills theory, with an abelian case, for which we obtain Maxwell-Chern-Simmons standard case.

1.2 Article structure

This article is structured as follows: A
1.- Introduction
2.- SU (2) calibration generators.
3.- Non-abelian calibration field.
4.- Curvature invariance.
5.- Yang-Mills approximation.
6.- Chern-Simmons actions and 2+1 reduction.
7.- Yang-Mills and Chern-Simmons action.
8.- Summary
9.- Conclusions on topological mass generation.
10.- Further work and paradigm.

ISSN: 2277-9655
Impact Factor: 5.164
CODEN: IJESS7

2. SU (2) CALIBRATION GENERATORS

Group generators are the startup Special unitary group is a unitary Lie group representing rotations in three-dimensional space-time. Its generators are Pauli matrices $\boldsymbol{\sigma^I}$ and $\boldsymbol{\sigma^J}$ and they satisfy following Lie Algebra:

$$[\sigma^I, \sigma^J] = 2ie^{IJk}\sigma^k \quad (1)$$

From now on *greek* inexes are used to represent external-space time coordinates, while *latin* indexes are used to represent coordinates of manifold internal spaces.

3. NON-ABELIAN CALIBRATION FIELD

One set A_μ as non-commutative calibration field. Let U be an element of special unitary group SU(2)that depends on each point of space time:

$$U(x) \ = \ e^{\frac{\sigma^I}{2i}g_1(x)} \quad (2)$$

And function $g_1(x)$ it's a regular position function over each point of space time generated.

a) Calibration transformation

Then, calibration field $A\mu$ transforms as shown next:

$$A_\mu \to A_\mu^{'} \quad (3)$$

With also:

$$A_\mu^{'} = UA_\mu U^{-1} + (\partial_\mu U)U^{-1} \quad (4)$$

Where a is *a* constant which takes values depending on U potential element and field features.

b) Calibration field expression

From calibration transform one may write calibration field expression as follows:

$$A_\mu \ = \ A_{\mu I}\frac{\sigma^I}{2i} \quad (5)$$

This calibration transform result is not exclusive of Chern-Simmons field theory, but from conformal field and scalar theory.

c) Field curvature

Field curvature from A_μ is written expressed as curvature is written in equation (6):

$$F_{\mu\nu}= \partial_\mu A_\nu - \partial_\nu A_\mu + [A\nu, A\mu] \quad (6)$$

From this curvature one may include Maxwell electromagnetic theory. Thus, we obtain following expression:

$$F\mu\nu = UF\mu\nu U^{-1} \quad (7)$$

4. CURVATURE INVARIANCE

For abelian Lie group cases calibration action lead to a field which curvature is invariant, such as U (1). In this paper, construction of calibration theory is done over special unitary group. SU (2) is not an abelian or commutative group, however it is possible to obtain a different kind of invariance from field curvature, even if field $A\mu$ comes from non-abelian Lie group.

Invariance for non-commutative Lie groups may be shown as covariance. To obtain an adequate and functional field theory it is indispensable to obtain invariance for Aμ curvature. From multi-linear algebra theory is possible to obtain invariant curvature from covariance terms. In relativistic formalism, one may compute the trace of curvature in order to obtain this convenient result. Calculating trace from terms of equation 7, and using cyclic property of trace we have:

$$Tr\,(F_{\mu\nu}F^{\mu\nu}) = Tr\,(UF_{\mu\nu}F^{\mu\nu}U^{-1}) \quad (8)$$

Trace value may be used to formulate Yang-Mills action.

5. YANG-MILLS APPROXIMATION

Let equation 8 be the integrand of a Yang-Mills action integral S_{YM} and integration done over 4-dimensional space d^4x one has:

$$SYM = \int Tr\,(F\mu\nu F^{\mu\nu})d^4x \quad (9)$$

Which is the same as:

$$SYM = \int Tr\,\,(UF\mu\nu F^{\mu\nu}U^{-1}) = Tr\,(F\mu\nu F^{\mu\nu}) \;(10)$$

Solving equation 10 to obtain motion equations we have:

$$D_\mu = \partial_\mu F^{\mu\,\nu} = 0 \qquad (11)$$

where covariant derivative is given by equation (12):

$$D_\mu = \partial_\mu + (A_\mu) \qquad (12)$$

Equation (12) may be used to build field equations of Yang Mills theory on unitary group.

6. CHERN-SIMMONS ACTION AND 2+1 REDUCTION

In order to reach 2+1 reduction from 4 dimensional Chern-Simmons action, we may obtain the invariant action S_{CS} building at first the integrand. We substitute equation (2) in (7), thus we obtain:

$$U\,F\mu\nu F^{\mu\nu}U^{-1} = ke^{\mu\nu\rho\sigma}F\mu\nu F\rho\sigma \;(13)$$

$$SCS = \int Tr\,(ke^{\mu\nu\rho\sigma}F\mu\nu F\rho\sigma)\,d^4x \quad (14)$$

And one can rewrite this expression as follows:

$$SCS = \int \partial_\mu Tr\,(ke^{\mu\nu\rho\sigma}A\nu\partial\rho A\sigma \;+\tfrac{2}{3}A\nu A\rho A\sigma)\,d^4x \quad (15)$$

Which is the same as equation 14 expressed in terms of total quadric-divergence. This fact is not relevant for the entire action, but it is possible to fix this term different from zero, such as it is done for abelian cases [16]. Then rewrote action is given by next equation:

$$SCS = \int Tr\,(ke^{\mu\nu\rho}A\nu\partial\rho A\sigma \;+\tfrac{2}{3}A\nu A\rho A\sigma)\,d^3x \qquad (16)$$

Which has the form of a Chern-Simmons 2+1 reduced action for a calibration over a scalar field (non-abelian in our case) field theory. Equation (16) represent action over a topological manifold within a scalar background field following potential form and transformation of equations (3-5).

7. YANG MILLS AND CHERN-SIMMONS ACTION

Chern-Simmons theory for non-abelian local groups may be compared with a Yang-Mills theory. At this point one may apply same methodology for Abelian symmetry local group cases [16]. Let S_{CS-YM} be Chern-Simmons and Yang-Mills action for non-abelian theory. Therefore, one may write it as:

$$S_{YM-CS} = \int \left((k_1 Tr\,(F\mu\nu F^{\mu\nu}) + k_2 Tr\left(e^{\mu\nu\rho}A\nu\partial_\rho A\sigma + \tfrac{2}{3}A\nu A\rho A\sigma\right)\right)d^3x \qquad (17)$$

From [14,15] one may write field equations for last action:

$$D_\mu F^{\mu\nu} - \frac{k_2}{4k_1}e^{\mu\nu\rho}F_{\rho\sigma} \quad (18)$$

ISSN: 2277-9655
Impact Factor: 5.164
CODEN: IJESS7

It is remarkable that the term $\frac{k_2}{4k_1}$ has mass units, this is topological mass term, which represents mass only induced by Chern-Simmons terms.

8. SUMMARY

Main ideas are written bellow. It is important to remark that non-abelian case is important to both physics and mathematics because its symmetric non-commutative nature. Choosing an adequate frame for dimensional reduction make a notable difference among two local symmetry Lie groups such as unitary group and special unitary group.

1.- Non-abelian field theory has been developed over SU (2) Lie group.
2.- It is not possible to obtain direct invariance of field curvature.
3.- It is possible to have covariance relation for field curvature calculating tensor trace on it.
4.- It is possible to obtain Yang-Mills action from this model.

5.- 2+1-dimensional reduction has been made thru Chern-Simmons non-abelian invariant.
6.- Yang-Mills and Chern-Simmons action has been formulated for this concrete example.
7.- Field equations for YM-CS field action are acquired.
8.- Topological mass has been recognized of YM-CS equations.

9. CONCLUSION FROM TOPOLOGICAL MASS ANALYSIS

It is interesting, that applied Chern-Simmons theory over invariance Lie local groups may generate topological mass. Regular case (abelian one) is described in [16], while a particular non-abelian case, has been described in this paper. Yang-Mills approach guarantees mass generation and Chern-Simmons action induce topological order on it. The fact that one can find topological mass generation in both, abelian and non-abelian Lie groups, lead us to think on a general formalism for topological mass, whether if field local Lie group theory is or it's not abelian. It is also necessary to remark relevancy on lower dimension manifold decomposition by 2+1 reduction and its importance in topological field theory.

10. FURTHER PARADIGM ON TOPOLOGICAL MASS GENERATION

Topological mass generation is an interesting phenomenon if we stand from a cosmological and physical point of view. Topological mass has been obtained many times thru its constant recognition of its distinguished units from Chern-Simmons action expressed in terms of mass units. Topological order has been questioned and studied may times by many physicists. For cosmologist who are specifically involved into development of topological field theory, cosmological order with topology and topological hierarchy of coupling constants, stablish topological order thru mass or energy (or even both) is important.

Further research in topological is about obtaining better and more concrete examples of topological mass, and specifically the acknowledge of topological quantities such as topological mass or energy. We would like to propose better field and cosmological models coming from topological interpretations.

REFERENCES

[1]. Physical origin of topological mass in 2+1 dimensions, D. Boyanovsky et. al, Stanford Linear Accelerator Center Stanford University, Nuclear Physics, Stanford, California, 94505, 1985, Opt key 3764.
[2]. Topological Mass Generation in Four Dimensions, Gia Dvalia, R. Jackiwb and So - Young Pic, Phys. Rev. Lett. 96, 081602 – Published 2 March 2006, https://doi.org/10.1103/PhysRevLett.96.081602.
[3]. Topological Mass Generation in Three-Dimensional String Theory, Nemanja Kaloper, Phys.Lett. B320 (1994) 16-20.
[4]. The Universe as a Set of Topological Fluids with Hierarchy and Fine Tuning of Coupling Constants in Terms of Graph Manifolds, Vladimir N. Efremov, Alfonso M. Hernandez Magdaleno, Fernando I. Becerra Lopez, arXiv:1309.0690[hep-th].
[5]. A. Smailagic and E. Spallucci, "Duality of massive gauge invariant theories in arbitrary space-time dimension," Phys. Rev. D61 (2000) 067701, hepth/9911089.
[6]. Faddeev-Jackiw analysis of topological mass-generating action, Chang-Yeong Leedag and Dong Won Leeddag, Journal of Physics A: Mathematical and General, Volume 31, Number 38.

ISSN: 2277-9655
Impact Factor: 5.164
CODEN: IJESS7

[7]. Anomalies and symmetry fractionalization in reflection-symmetric topological order, Ethan Lake Phys. Rev. B 94, 205149 – Published 30 November 2016. -[8] Topology and mass generation mechanisms in abelian gauge field theories, Bruno Bertrand, Facult´e des Sciences D´epartement de Physique, Center for Particle Physics and Phenomenology, (Ph.D. thesis)- 2008.

[8]. Canonical quantization of (2+1)-dimensional QED with a topological mass term, Kurt Haller and Edwin Lim-Lombridas, Phys. Rev. D 46, 1737 – Published 15 August 1992, 10.1103/PhysRevD.46.1737, Phys.Rev. D46 (1992) 17371749.

[9]. Three-Dimensional Massive Gauge Theories, Stanley Deser (Brandeis U.), R. Jackiw, S. Templeton (MIT, LNS), Phys.Rev.Lett. 48 (1982) 975-978, DOI:10.1103/PhysRevLett.48.975, 1982 - 4 pages.

[10]. B. Bertrand, "The dual formulations of abelian Higgs models and their generalisations : A zoo of topological defect solutions." Internal note, january, 2008.

[11]. J. Hong, Y. Kim, and P. Y. Pac, "Multivortex solutions of the Abelian Chern-Simons-Higgs theory," Phys. Rev. Lett. 64 (1990) 2230.

[12]. E. Harikumar and M. Sivakumar, "On the equivalence between topologically and non-topologically massive abelian gauge theories," Mod. Phys. Lett. A15 (2000) 121–132, hep-th/9911244.

[13]. O. A. Tretyakov † and O. Akgun. Derivation of klein-gordon equation from maxwell's equations and study of relativistic time-domain waveguide modes, Progress In Electromagnetics Research, Vol. 105, 171–191, 2010.

[14]. Topological mass generation four-dimensional gauge theory, G. Savvidy, Physics Letters B, Volume 694, Issue 1, 25 October 2010, Pages 65-73.

[15]. Abelian calibration field theory on U (1) Lie group with Lorenz invariants in 2+1 dimensions with topological mass generation. International Journal of engineering sciences and research technology. ISSN: 2277-9655, JESRT: 7(10), October, 2018, Alatorre* et al., 7(10): October, 2018.